18.06 MIDTERM 1

October 2, 2019 (50 minutes)

Please turn cell phones off completely and put them away.

No books, notes, or electronic devices are permitted during this exam.

You must show your work to receive credit. **JUSTIFY EVERYTHING**.

Please write your name on **ALL** pages that you want graded (those will be the ones we scan).

The back sides of the paper will **NOT** be graded (for scratch work only).

Do not unstaple the exam, nor reorder the sheets. If you need more space, there are extra sheets at the end of this book, but write the problem number at the top of each.

Problem 1 has $\boxed{3}$ parts, Problem 2 has $\boxed{4}$ parts, Problem 3 has $\boxed{4}$ parts.

NAME: ______________________________

MIT ID NUMBER: ______________________________

RECITATION INSTRUCTOR: ______________________________

SCRATCH WORK

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PROBLEM 1 **NAME:** ____________________

(1) Use Gaussian elimination to write the matrix $A = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 \\ -1 & 2 & -2 & -1 & 2 \\ 0 & -2 & 0 & 0 & -1 \\ 0 & 0 & 0 & -2 & 2 \end{bmatrix}$ as $A = LU$, where L is a lower triangular 4×4 matrix and U is a 4×5 matrix in row echelon form.

(15 pts)

SCRATCH WORK

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NAME: ____________________

(2) Express the matrix L in part (1) as a product of elimination matrices $E_{ij}^{(\lambda)}$ for various $i > j$ and constants λ. Then do the same for its inverse L^{-1}. *(10 pts)*

(3) With the same notations as in part (1), find a permutation matrix P such that $PA' = LU$, where $A' = \begin{bmatrix} -1 & 2 & -2 & -1 & 2 \\ 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -2 & 2 \\ 0 & -2 & 0 & 0 & -1 \end{bmatrix}$. *(5 pts)*

SCRATCH WORK

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PROBLEM 2 **NAME:** ____________________

Consider the system of equations:

$$\begin{cases} a - 3b + 6c - d = 1 \\ -2a + 5b - 11c + 2d = -2 \end{cases} \qquad (*)$$

(1) Write the system as $A \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \mathbf{b}$ for a suitably chosen 2×4 matrix A and 2×1 vector $\mathbf{b}$.

(5 pts)

(2) Use Gauss-Jordan elimination to put A in reduced row echelon form. *(10 pts)*

SCRATCH WORK

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NAME: ____________________

(3) What are basis vectors for the nullspace of A? What is its dimension? *(10 pts)*

(4) What is the general (i.e. complete) solution of the system $(*)$? *(10 pts)*

SCRATCH WORK

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PROBLEM 3 **NAME:** ____________________

Consider the matrix $B = \begin{bmatrix} 1 & 1 & 5 \\ 2 & 0 & 4 \\ 3 & 1 & 9 \end{bmatrix}$.

(1) Put the matrix in row echelon form, and compute a basis for its column space. *(15 pts)*

SCRATCH WORK

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NAME: ____________________

(2) Find a linear combination of the columns of B which is 0. *(5 pts)*

(3) Compute the matrix $S = B^T B$ <u>and</u> the difference $S^T - S$. *(10 pts)*

(4) Explain why for any 3×3 matrix X, the product XB cannot be invertible. *(5 pts)*

SCRATCH WORK

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Continuation of work on problem number _____ **NAME:** ____________________

SCRATCH WORK

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SCRATCH WORK

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